

# INTEGRAL TAK TENTU (ITT)

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LOGO

## DEFINISI & SIFAT ITT

$$\int f(x) dx = F(x) + C$$

↙
↓
↘  
 Integrand      Fungsi primitif      konstanta integrasi

### SIFAT-SIFAT ITT:

1.  $\int kf(x) dx = k \int f(x) dx$ ,  $k$ : konstanta
2.  $\int \{f(x) \pm g(x)\} dx = \int f(x) dx \pm \int g(x) dx$

## Beberapa rumus ITT

1.  $\int 0 dx = C$
2.  $\int x^n dx = \frac{1}{n+1} x^{n+1} + C; (n \neq -1)$
3.  $\int \cos x dx = \sin x + C$
4.  $\int \sin x dx = -\cos x + C$
5.  $\int \sec^2 x dx = \operatorname{tg} x + C$
6.  $\int \cos ec^2 x dx = -\cot g x + C$
7.  $\int \sec x \operatorname{tg} x dx = \sec x + C$
8.  $\int \cos ec x \cot g x dx = -\cos ec x + C$
9.  $\int e^x dx = e^x + C$
10.  $\int \frac{1}{x} dx = \ln x + C$
11.  $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x + C$
12.  $\int \frac{1}{1+x^2} dx = \operatorname{arctg} x + C$

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## Contoh:

Selesaikan:  $I = \int \frac{1}{x^3} dx$

Penyelesaian:  $I = \int x^{-3} dx = \frac{1}{-3+1} x^{-3+1} + C = -\frac{1}{2} x^{-2} + C$

Selesaikan:  $I = \int x(1-x^2)^5 dx$

Penyelesaian:

Misal:  $u = 1 - x^2 \rightarrow \frac{du}{dx} = -2x \rightarrow x dx = -\frac{1}{2} du$

$I = \int u^5 \left(-\frac{1}{2} du\right) = -\frac{1}{2} \cdot \frac{1}{6} u^6 + C = -\frac{1}{12} (1-x^2)^6 + C$

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## Integrasi Parsial

$$\int u dv = uv - \int v du$$

Rumus ini sangat berguna terutama jika integrand terdiri dari fungsi-fungsi transcendent, misalnya:  $\ln x$ ,  $\arcsin x$ ,  $\arctg x$ , atau hasil ganda seperti:  $xe^x$ ,  $e^x \sin x$ ,  $x \cos x$ ,  $x^2 \ln x$ .

Cara memakai rumus ini:

- $dv$  dipilih sehingga  $v$  mudah dicari
- $\int v du$  harus menjadi lebih mudah daripada  $\int u dv$ .

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## Contoh:

$$1. I = \int xe^{-x} dx$$

$$\text{Misal: } u = x$$

$$du = dx$$

$$dv = e^{-x} dx$$

$$v = \int e^{-x} dx = -e^{-x}$$

$$I = xe^{-x} - \int -e^{-x} dx = xe^{-x} - e^{-x} + C.$$

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$$2. I = \int \operatorname{arctg} x \, dx$$

$$\text{Misal: } u = \operatorname{arctg} x$$

$$dv = dx$$

$$du = \frac{1}{1+x^2}$$

$$v = \int dx = x$$

$$I = \int \operatorname{arctg} x \, dx = x \operatorname{arctg} x - \int x \cdot \frac{1}{1+x^2} \, dx$$

$$= x \operatorname{arctg} x - \frac{1}{2} \int \frac{2x}{1+x^2} \, dx$$

$$= x \operatorname{arctg} x - \frac{1}{2} \ln(1+x^2) + C$$

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## Penerapan 1/D pada ITT

$$D = \frac{d}{dx} \text{ operator derivatif/turunan}$$

$$\frac{1}{D} = D^{-1} = \int \dots dx \text{ operator integral}$$

Deret Maclaurin dari:

$$\frac{1}{1-D} = 1 + D + D^2 + \dots$$

$$\frac{1}{1+D} = 1 - D + D^2 - \dots$$

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$$D \sin ax = a \cos ax, \quad D^2 \sin ax = -a^2 \sin ax$$

$$D \cos ax = -a \sin ax, \quad D^2 \cos ax = -a^2 \cos ax$$

$$\left. \begin{aligned} (D^2 - b^2) \sin ax &= (-a^2 - b^2) \sin ax \\ (D^2 - b^2) \cos ax &= (-a^2 - b^2) \cos ax \end{aligned} \right\} \text{sehingga:}$$

$$\frac{1}{D^2 - b^2} \sin ax = \frac{\sin ax}{-a^2 - b^2}; \quad \frac{1}{D^2 - b^2} \cos ax = \frac{\cos ax}{-a^2 - b^2}.$$

$$\text{RUMUS: } \frac{1}{D} e^{ax} V = e^{ax} \frac{1}{D+a} V$$

$$\frac{1}{D} UV = U \cdot \frac{1}{D} V - DU \cdot \frac{1}{D^2} V + D^2 U \cdot \frac{1}{D^3} V - \dots$$

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### Contoh 1:



$$I = \int e^{3x} (x^2 - 3x + 2) dx = \frac{1}{D} e^{3x} (x^2 - 3x + 2) = e^{3x} \frac{1}{D+3} (x^2 - 3x + 2)$$

$$= e^{3x} \cdot \frac{1}{3} \cdot \frac{1}{\left(1 + \frac{D}{3}\right)} (x^2 - 3x + 2) = \frac{1}{3} e^{3x} \left(1 - \frac{D}{3} + \frac{D^2}{9} - \dots\right) (x^2 - 3x + 2)$$

$$= \frac{1}{3} e^{3x} \left( (x^2 - 3x + 2) - \frac{1}{3} (2x - 3) + \frac{1}{9} \cdot 2 \right) + C = \frac{1}{3} e^{3x} \left( x^2 - \frac{11}{3} x + \frac{29}{9} \right) + C$$

$$I = \frac{1}{27} e^{3x} (9x^2 - 33x + 29) + C$$

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## Contoh 2:

$$\begin{aligned}
 \int x^3 e^{-2x} dx &= \frac{1}{D} x^3 e^{-2x} = x^3 \cdot \frac{1}{D} e^{-2x} - Dx^3 \cdot \frac{1}{D^2} e^{-2x} + D^2 x^3 \cdot \frac{1}{D^3} e^{-2x} - D^3 x^3 \cdot \frac{1}{D^4} e^{-2x} \\
 &= -\frac{1}{2} x^3 e^{-2x} - \frac{3}{4} x^2 e^{-2x} - \frac{3}{4} x e^{-2x} - \frac{3}{8} e^{-2x} + C \\
 &= -\frac{1}{8} e^{-2x} (4x^3 + 6x^2 + 6x + 3) + C
 \end{aligned}$$

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## Rumus Reduksi

1.  $\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$ ,  $n$  : bilangan bulat positif  $\geq 1$ .
2.  $\int \sin^n x dx = \frac{-\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$ ,  $n$  : bilangan bulat positif  $\geq 2$ .
3.  $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$ ,  $n$  : bilangan bulat positif  $\geq 2$ .
4.  $\int \operatorname{tg}^n x dx = \frac{\operatorname{tg}^{n-1} x}{n-1} - \frac{n-1}{n} \int \operatorname{tg}^{n-2} x dx$ ,  $n$  : bilangan bulat positif  $\geq 2$ .

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$$5. \int \cotg^n x dx = \frac{-\cotg^{n-1} x}{n-1} - \int \cotg^{n-2} x dx; \quad n : \text{bilangan bulat positif } \geq 2.$$

$$6. \int \sec^n x dx = \frac{\sec^{n-2} x \operatorname{tg} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx; \quad n : \text{bilangan bulat positif } \geq 2.$$

$$7. \int \operatorname{cosec}^n x dx = \frac{-\operatorname{cosec}^{n-2} x \cotg x}{n-1} + \frac{n-2}{n-1} \int \operatorname{cosec}^{n-2} x dx; \quad n : \text{bilangan bulat positif } \geq 2$$

$$8. \int \frac{dx}{(1+x^2)^n} = \frac{x}{2(n-1)(1+x^2)^{n-1}} + \frac{2n-3}{2n-2} \int \frac{dx}{(1+x^2)^{n-1}}; \quad n \neq 1.$$

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## Integrasi Fungsi Pecah Rasional

$$\int \frac{T(x)}{N(x)} dx$$

dengan derajat pembilang < derajat penyebut.

Ada beberapa kasus berhubungan dengan penyebut:

1. Jika  $N(x) = (ax+b)(cx+d)$  maka

$$\frac{T(x)}{N(x)} = \frac{A}{ax+b} + \frac{B}{cx+d}$$

2. Jika  $N(x) = (ax+b)^k (cx+d)$  maka

$$\frac{T(x)}{N(x)} = \frac{A}{ax+b} + \frac{B}{(ax+b)^2} + \dots + \frac{K}{(ax+b)^k} + \frac{U}{cx+d}$$

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3. Jika  $N(x) = (ax^2 + bx + c)(px + q)$  dengan  $D = (b^2 - 4ac) < 0$  maka

$$\frac{T(x)}{N(x)} = \frac{Ax + B}{ax^2 + bx + c} + \frac{C}{px + q}$$

4. Jika  $N(x) = (ax^2 + bx + c)^2(px + q)$  dengan  $D = (b^2 - 4ac) < 0$  maka

$$\frac{T(x)}{N(x)} = \frac{Ax + B}{ax^2 + bx + c} + \frac{Cx + D}{(ax^2 + bx + c)^2} + \dots + \frac{Ux + V}{(ax^2 + bx + c)^k} + \frac{W}{px + q}$$

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### Contoh 1:

$$I = \int \frac{23 - 2x}{2x^2 + 9x - 5} dx = \int \frac{23 - 2x}{(2x - 1)(x + 5)} dx$$

$$\frac{23 - 2x}{(2x - 1)(x + 5)} = \frac{A}{2x - 1} + \frac{B}{x + 5} \text{ dikalikan dengan } (2x - 1)(x + 5), \text{ menjadi}$$

$$23 - 2x = A(x + 5) + B(2x - 1)$$

$$\text{Untuk } x = -5 \Rightarrow 23 - 2(-5) = B(-11) \Rightarrow B = -3$$

$$\text{Untuk } x = \frac{1}{2} \Rightarrow 23 - 2\left(\frac{1}{2}\right) = A\left(\frac{11}{2}\right) \Rightarrow A = 4$$

$$I = \int \frac{4}{2x - 1} dx + \int \frac{-3}{x + 5} dx = 2 \ln|2x - 1| - 3 \ln|x + 5| + C$$

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## Contoh 2:

$$I = \int \frac{3x+1}{x^2+2x+1} dx = \int \frac{3x+1}{(x+1)^2} dx$$

$$\frac{3x+1}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} \text{ dikalikan dengan } (x+1)^2, \text{ menjadi}$$

$$3x+1 = A(x+1) + B$$

$$\text{Untuk } x = -1 \Rightarrow -3+1 = B \Rightarrow B = -2$$

$$\text{Untuk } x = 0 \Rightarrow 1 = A+B \Rightarrow A = 3$$

$$I = \int \frac{3}{x+1} dx + \int \frac{-2}{(x+1)^2} dx = 3 \ln|x+1| + \frac{2}{x+1} + C$$

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## Contoh 3:

$$I = \int \frac{x-1}{(x+1)(x^2+1)} dx$$

$$\frac{x-1}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1} \text{ dikalikan } (x+1)(x^2+1), \text{ menjadi}$$

$$x-1 = A(x^2+1) + (Bx+C)(x+1)$$

$$\text{Ketemu } A = -1, B = 1, C = 0$$

$$I = \int \frac{-1}{x+1} dx + \int \frac{x}{x^2+1} dx = -\ln|x+1| + \frac{1}{2} \ln|x^2+1| + C$$

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### Contoh 4:

$$I = \int \frac{2x^2 + 3}{(x^2 + 1)^2} dx$$

$$\frac{2x^2 + 3}{(x^2 + 1)^2} = \frac{Ax + B}{(x^2 + 1)} + \frac{Cx + D}{(x^2 + 1)^2} \text{ dikalikan dengan } (x^2 + 1)^2, \text{ menjadi}$$

$$2x^2 + 3 = (Ax + B)(x^2 + 1) + Cx + D$$

$$\left. \begin{array}{l} x=0 \Rightarrow 3 = B + D \\ \text{Untuk } x=1 \Rightarrow 5 = 2A + 2B + C + D \\ x=-1 \Rightarrow 5 = -2A + 2B - C + D \\ x=2 \Rightarrow 11 = 10A + 5B + 2C + D \end{array} \right\} \Rightarrow B = 2, D = 1, A = 0, C = 0.$$

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$$I = \int \left[ \frac{2}{x^2 + 1} + \frac{1}{(x^2 + 1)^2} \right] dx = 2 \int \frac{1}{1 + x^2} dx + \int \frac{1}{(1 + x^2)^2} dx$$

$$= 2 \operatorname{arctg} x + \left\{ \frac{x}{2(1 + x^2)} + \frac{1}{2} \operatorname{arctg} x \right\} + C$$

$$= \frac{5}{2} \operatorname{arctg} x + \frac{x}{2(1 + x^2)} + C$$

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## Integrasi Fungsi Trigonometri

Ingat-ingat beberapa rumus berikut:

- $\sin^2 x + \cos^2 x = 1$
- $\sin 2x = 2 \sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(-x) = -\sin x$ ,  $\cos(-x) = \cos x$
- $\sin \alpha x \sin \beta x = \frac{1}{2}[\sin(\alpha + \beta)x + \sin(\alpha - \beta)x]$
- $\cos \alpha x \cos \beta x = \frac{1}{2}[\cos(\alpha + \beta)x + \cos(\alpha - \beta)x]$
- $\sin \alpha x \sin \beta x = \frac{1}{2}[\cos(\alpha - \beta)x - \cos(\alpha + \beta)x]$

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**Bentuk:**  $\int \sin \alpha x \cos \beta x dx$ ;  $\int \cos \alpha x \cos \beta x dx$ ;  $\int \sin \alpha x \sin \beta x dx$

- $\int \sin 6x \cos 2x dx = \int \frac{1}{2}[\sin 8x + \sin 4x] dx = \frac{1}{2} \left[ -\frac{1}{8} \cos 8x - \frac{1}{4} \cos 4x \right] + C$
- $\int \cos 6x \cos 3x dx = \int \frac{1}{2}[\cos 9x + \cos 3x] dx = \frac{1}{2} \left[ \frac{1}{9} \sin 9x + \frac{1}{3} \sin 3x \right] + C$
- $\int \cos^2 2x dx = \int \frac{1}{2}[1 + \cos 4x] dx = \frac{1}{2} \left[ x + \frac{1}{4} \sin 4x \right] + C$
- $\int \sin 3x \sin 2x dx = \int \frac{1}{2}[\cos x - \cos 5x] dx = \frac{1}{2} \left[ \sin x - \frac{1}{5} \sin 5x \right] + C$

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**Bentuk:**  $\int R(\sin x, \cos x) dx$ ;  $R =$  fs rasional

Substitusi:  $\operatorname{tg} \frac{x}{2} = t \Rightarrow \frac{x}{2} = \operatorname{arctg} t, \quad x = 2 \operatorname{arctg} t \Rightarrow dx = \frac{2dt}{1+t^2}$

$$\sin x = \frac{2 \sin \frac{x}{2} \cos \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} = \frac{2 \operatorname{tg} \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} \Rightarrow \sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} = \frac{1 - \operatorname{tg}^2 \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} \Rightarrow \cos x = \frac{1-t^2}{1+t^2}$$

Maka:

$$\int R(\sin x, \cos x) dx = \int R\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right) \cdot \frac{2dt}{1+t^2}$$

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**Contoh:**

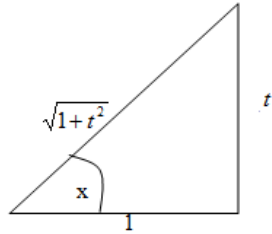
$$\int \frac{dx}{5+4\cos x} = \int \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)} \cdot \frac{2dt}{1+t^2} = 2 \int \frac{dt}{9+t^2} = \frac{2}{3} \operatorname{arctg} \frac{t}{3} + C$$

$$= \frac{2}{3} \operatorname{arctg} \left( \frac{1}{3} \operatorname{tg} \frac{x}{2} \right) + C$$

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**Bentuk:**  $\int R(\sin x, \cos x) dx = \int R(-\sin x, -\cos x) dx$

Substitusi:  $\operatorname{tg} x = t \Rightarrow x = \operatorname{arctg} t \Rightarrow dx = \frac{dt}{1+t^2}$



$$\sin x = \frac{t}{\sqrt{1+t^2}}$$

$$\cos x = \frac{1}{\sqrt{1+t^2}}$$

CONTOH:

$$I = \int \frac{1}{\sin^2 x - \sin x \cos x} dx = \int \frac{1}{\left(\frac{t}{\sqrt{1+t^2}}\right)^2 - \left(\frac{t}{\sqrt{1+t^2}}\right)\left(\frac{1}{\sqrt{1+t^2}}\right)} \cdot \frac{dt}{1+t^2}$$

$$= \int \frac{dt}{t(t-1)} = \int \left(\frac{1}{t-1} - \frac{1}{t}\right) dt = \ln \left| \frac{t-1}{t} \right| + C = \ln |1 - \cot g x| + C$$

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**Bentuk:**  $\int R(\operatorname{tg} x) dx$

Substitusi:  $\operatorname{tg} x = t \Rightarrow x = \operatorname{arctg} t \Rightarrow dx = \frac{dt}{1+t^2}$

CONTOH:

$$I = \int \frac{1+\operatorname{tg} x}{1-\operatorname{tg} x} dx = \int \frac{1+t}{1-t} \cdot \frac{dt}{1+t^2} = \int \frac{1+t}{(1-t)(1+t^2)} dt$$


$$\frac{1+t}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2} \text{ dikalikan dengan } (1-t)(1+t^2), \text{ maka:}$$

$$1+t = A(1+t^2) + (Bt+C)(1-t)$$

$$1+t = (A+C) + (B-C)t + (A-B)t^2$$

$$\left. \begin{array}{l} A+C=1 \\ B-C=1 \\ A-B=0 \end{array} \right\} \Rightarrow A=1, B=1, C=0.$$

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$$I = \int \left[ \frac{1}{1-t} + \frac{t}{1+t^2} \right] dt = -\ln|1-t| + \frac{1}{2} \ln|1+t^2| + C = -\ln \left| \frac{1-t}{\sqrt{1+t^2}} \right| + C$$

$$= -\ln \left| \frac{1 - \operatorname{tg} x}{\sec x} \right| + C = -\ln|(1 - \operatorname{tg} x) \cos x| + C = -\ln|\cos x - \sin x| + C$$

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## Integrasi dgn substitusi trigonometri



No	Integrand	Substitusi	Hasil:
1	$\int \frac{dx}{\sqrt{a^2 - x^2}}$	$x = a \sin t$	$\arcsin \frac{x}{a} + C$
2	$\int \sqrt{a^2 - x^2} dx$		$\frac{1}{2} \left( x\sqrt{a^2 - x^2} + a^2 \arcsin \frac{x}{a} \right) + C$
3	$\int \frac{dx}{a^2 + x^2}$	$x = a \operatorname{tg} t$	$\frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$
4	$\int \frac{dx}{\sqrt{a^2 + x^2}}$		$\ln \left  x + \sqrt{x^2 + a^2} \right  + C$

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5	$\int \sqrt{x^2 + a^2} dx$		$\frac{1}{2} \left( x\sqrt{x^2 + a^2} + a^2 \ln  x + \sqrt{x^2 + a^2}  \right) + C$
6	$\int \frac{dx}{\sqrt{x^2 - a^2}}$	$x = a \sec t$	$\ln  x + \sqrt{x^2 - a^2}  + C$
7	$\int \sqrt{x^2 - a^2} dx$		$\frac{1}{2} \left( x\sqrt{x^2 - a^2} - a^2 \ln  x + \sqrt{x^2 - a^2}  \right) + C$

$$1. \int \frac{1}{\sqrt{a^2 - x^2}} dx = \int \frac{1}{a \cos t} \cdot a \cos t dt = \int dt = t + C = \arcsin \frac{x}{a} + C$$