BILANGAN KOMPLEKS

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Bilangan Kompleks: $Z = x + yi, \quad i = \sqrt{-1}$

Misal: $Z_1 = a + bi,\ Z_2 = c + di$

Operasi pada Bilangan Kompleks:

1. Penjumlahan

   $$Z_1 + Z_2 = (a + c) + (b + d)i$$

2. Pengurangan

   $$Z_1 - Z_2 = (a - c) + (b - d)i$$

3. Perkalian

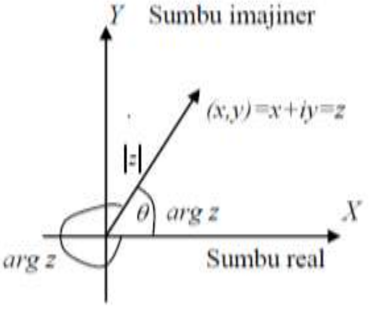
$$Z_1 Z_2 = (a+bi)(c+di)$$
$$= ac + adi + bci + bdi^2$$
$$= (ac - bd) + (ad + bc)i$$

4. Pembagian

$$\frac{Z_1}{Z_2} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di}$$
$$= \frac{ac - adi + bci - bdi^2}{c^2 - d^2 i^2}$$
$$= \frac{(ac+bd) + (bc-ad)i}{c^2 + d^2}$$

5. Konjuget : $\overline{Z}_1 = a - bi$

Bentuk Kutub



$$Z = x + yi, \quad i = \sqrt{-1}$$
$$Modulus : |Z| = r = \sqrt{x^2 + y^2}$$
$$\theta = \arg Z$$
$$\left.\begin{array}{l} \sin\theta = \dfrac{y}{r} \\ \cos\theta = \dfrac{x}{r} \end{array}\right\} \theta = ...$$
$$Z = r\ (\cos\theta + i \sin\theta)$$
$$= r\ cis\,\theta$$

Operasi dlm bentuk kutub

$$Z_1 Z_2 = [r_1(\cos\theta_1 + i\sin\theta_1)][r_2(\cos\theta_2 + i\sin\theta_2)]$$
$$= r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$
$$\frac{Z_1}{Z_2} = \frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)}$$
$$= \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

Theorema de Moivre:

$$[r(\cos\theta + i\sin\theta)]^n = r^n[\cos n\theta + i\sin n\theta]$$

Akar dari Bilangan Kompleks

$$Z^n = x + yi, \quad n = 1,2,3......$$
$$Z_{1,2,3...n} = \sqrt[n]{x+yi} = (x+yi)^{1/n}$$
$$= [r(\cos\theta + i\sin\theta)]^{1/n}$$
$$= r^{1/n}[\cos(\theta + k.360^o) + i\sin(\theta + k.360^o)]^{1/n}$$
$$= r^{1/n}\left[\cos\left(\frac{\theta + k..360^o}{n}\right) + i\sin\left(\frac{\theta + k..360^o}{n}\right)\right], \; k = 0,1,2,3....$$

Contoh 1:

Ubahlah Z=1-i dalam bentuk kutub

Penyelesaian:

$$x = 1, y = -1$$

$$r = \sqrt{x^2 + y^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

Kwadran IV

$$\left.\begin{aligned} \sin\theta &= \frac{-1}{\sqrt{2}} = -\frac{1}{2}\sqrt{2} \\ \cos\theta &= \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{2} \end{aligned}\right\} \theta = 360^o - 45^o = 315^o.$$

$$\therefore Z = r\ (\cos\theta + i\sin\theta)$$

$$= \sqrt{2}(\cos 315^o + i\sin 315^o)$$

Contoh 2: Dapatkan akar-akar dari $Z^3 = i - \sqrt{3}$

Penyelesaian:

$$Z_{1,2,3} = \sqrt[3]{i - \sqrt{3}};$$

$$= r^{1/3}\left[\cos\left(\frac{\theta + k.360^o}{3}\right) + i\sin\left(\frac{\theta + k.360^o}{3}\right)\right]$$

$$x = -\sqrt{3},\ \ y = 1$$

$$r = \sqrt{\left(-\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$$

$$\left.\begin{aligned} \sin\theta &= \frac{y}{r} = \frac{1}{2} \\ \cos\theta &= \frac{x}{r} = -\frac{1}{2}\sqrt{3} \end{aligned}\right\} \theta = 180^o - 30^o = 150^o$$

$$k = 0 \rightarrow Z_1 = 2^{1/3}\left[\cos\left(\frac{150^o + 0.360^o}{3}\right) + i\sin\left(\frac{150^o + 0.360^o}{3}\right)\right]$$

$$= \sqrt[3]{2}\left[\cos 50^o + i\sin 50^o\right]$$

$$k = 1 \rightarrow Z_2 = 2^{1/3}\left[\cos\left(\frac{150^o + 1.360^o}{3}\right) + i\sin\left(\frac{150^o + 1.360^o}{3}\right)\right]$$

$$= \sqrt[3]{2}\left[\cos 170^o + i\sin 170^o\right]$$

$$k = 2 \rightarrow Z_3 = 2^{1/3}\left[\cos\left(\frac{150^o + 2.360^o}{3}\right) + i\sin\left(\frac{150^o + 2.360^o}{3}\right)\right]$$

$$= \sqrt[3]{2}\left[\cos 290^o + i\sin 290^o\right]$$