

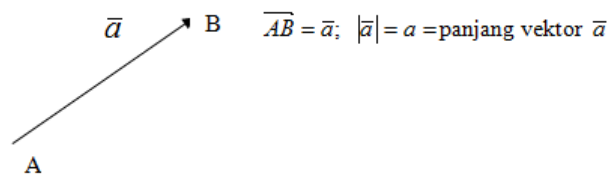
Logo

ALJABAR VEKTOR

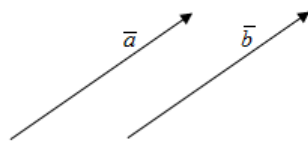
Anita T. Kurniawati

DEFINISI

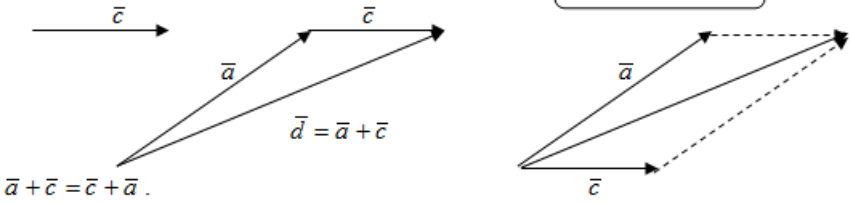
- Vektor: kuantiti yang punya besar dan arah.
- Skalar: kuantiti yang punya besar.



Dua vektor dikatakan sama ($\bar{a} = \bar{b}$) jika searah dan sama panjang.



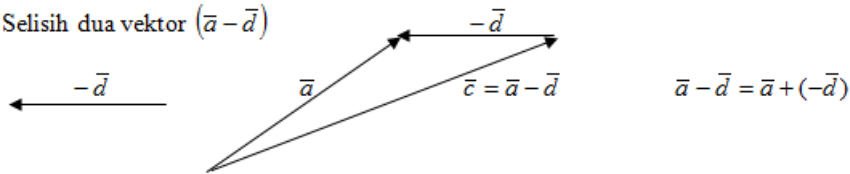
Jumlah dua vektor ($\vec{a} + \vec{c}$)



$\vec{d} = \vec{a} + \vec{c}$

$\vec{a} + \vec{c} = \vec{c} + \vec{a}$

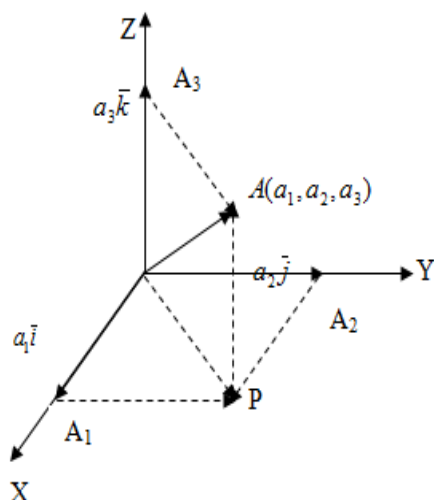
Selisih dua vektor ($\vec{a} - \vec{d}$)



$\vec{c} = \vec{a} - \vec{d}$

$\vec{a} - \vec{d} = \vec{a} + (-\vec{d})$

Komponen suatu vektor



$\vec{OA} = \vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

P proyeksi A pada bidang XOY

$\vec{OA} = a_1\vec{i}, \vec{OA}_2 = a_2\vec{j}, \vec{OA}_3 = a_3\vec{k} = \vec{PA}$

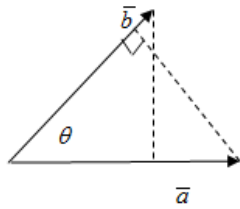
$\vec{OP} = \vec{OA}_1 + \vec{OA}_2 = a_1\vec{i} + a_2\vec{j}$

$\vec{a} = \vec{OA} = \vec{OP} + \vec{PA} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

↓

$a = |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Perkalian Titik (DOT PRODUCT)



Definisi: $\vec{a} \cdot \vec{b} = ab \cos \theta$, $(0 \leq \theta \leq \pi)$

$$\theta = \angle(\vec{a}, \vec{b}); \quad \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}.$$

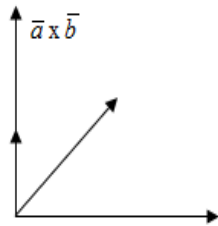
$$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1.$$

$$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0.$$

$$\left. \begin{array}{l} \vec{a} = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} \\ \vec{b} = b_1\vec{i} + b_2\vec{j} + b_3\vec{k} \end{array} \right\} \Rightarrow \vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{ab} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Perkalian Silang (CROSS PRODUCT)



Definisi: $\vec{a} \times \vec{b} = (a b \sin \theta) \vec{e}$; $(0 \leq \theta \leq \pi)$

$\theta = \angle(\vec{a}, \vec{b})$ diukur dari \vec{a} ke \vec{b} ;

\vec{e} : vektor satuan yang tegak lurus bidangnya \vec{a} dan \vec{b} .

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0; \quad \vec{i} \times \vec{j} = \vec{k}; \quad \vec{j} \times \vec{k} = \vec{i}; \quad \vec{k} \times \vec{i} = \vec{j}; \quad \vec{j} \times \vec{i} = -\vec{k}; \quad \vec{k} \times \vec{j} = -\vec{i}; \quad \vec{i} \times \vec{k} = -\vec{j}.$$

$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a}).$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}.$$

Luas jajaran genjang yang dibentuk \vec{a} dan \vec{b} , adalah:

$$L = |\vec{a} \times \vec{b}| = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Luas segitiga yang dibentuk \vec{a} dan \vec{b} :

$$L_{\Delta} = \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Volume balok miring (Paralelepipedum) dengan sisi-sisi $\vec{a}, \vec{b}, \vec{c}$:

$$V = |\vec{a} \cdot (\vec{b} \times \vec{c})| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$